# Numbers

COMP370

Introduction to Computer Architecture

## Goals

- Understand binary and hexadecimal numbers
- Be able to convert between number bases
- Understand binary fractions

## Unary Numbers

| Decimal | Unary |
|---|---|
| 1 | \| |
| 2 | \|\| |
| 3 | \|\|\| |
| 4 | \|\|\|\| |
| 5 | 𝍸 |
| 6 | 𝍸 \| |

## Decimal Numbers

- Decimal numbers work well for humans.
- Each position is 10 times the one to the right

3,148 is

| 1000's | 100's | 10's | 1's |
|---|---|---|---|
| 3 | 1 | 4 | 8 |

## Binary Numbers

- Computers usually use binary numbers, base 2
- Binary numbers only have two digits, 0 and 1
- Each position is 2 times the one to the right

$010011_2$ ($19_{10}$) is

| 32's | 16's | 8's | 4's | 2's | 1's |
|---|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 1 | 1 |

## Hexadecimal Numbers

- It can be convenient to represent values in base 16 or hexadecimal.
- Hex numbers have the digits

0 1 2 3 4 5 6 7 8 9 A B C D E F

- Each position is 16 times the one to the right

$31E8_{16}$ ($12{,}776_{10}$) is

| 4096's | 256's | 16's | 1's |
|---|---|---|---|
| 3 | 1 | E | 8 |

## Indicating the Number Base

- The number or radix base can be specified by a subscript after the number.

$47_{10}$ $101111_2$ $2F_{16}$

- Computer languages use a prefix

47 0B101111 0X2F

- Binary starts with zero "B" *(C++ only)*
- Hexadecimal starts with zero "X"

## Converting Decimal to Binary

- Start with the largest even power of 2 bigger than the decimal number
- For each power of two

```
if the number >= power of 2 {
    put a 1 bit to the right;
    Subtract power of 2 from the number;
} else {
    put a 0 to the right;
}
```

- Repeat for all powers of 2

4096
2048
1024
512
256
128
64
32
16
8
4
2
1

| | | |
|---|---|---|
| 4096 | 0 | 317 |
| 2048 | 0 | 317 |
| 1024 | 0 | 317 |
| 512 | 0 | 317 |
| 256 | 1 | 61 |
| 128 | 0 | 61 |
| 64 | 0 | 61 |
| 32 | 1 | 29 |
| 16 | 1 | 13 |
| 8 | 1 | 5 |
| 4 | 1 | 1 |
| 2 | 0 | 1 |
| 1 | 1 | 1 |

## Decimal to Binary Example

- Convert 317 to binary
- The answer is

0000100111101

## Decimal-to-Binary Conversion

Convert the following decimal numbers to binary:

$12 = 8 + 4 = 2^3 + 2^2$ → 1100

$25 = 16 + 8 + 1 = 2^4 + 2^3 + 2^0$ → 11001

$58 = 32 + 16 + 8 + 2 = 2^5 + 2^4 + 2^3 + 2^0$ → 111010

$82 = 64 + 16 + 2 = 2^6 + 2^4 + 2^0$ → 1010010

## Repeated Division Conversion

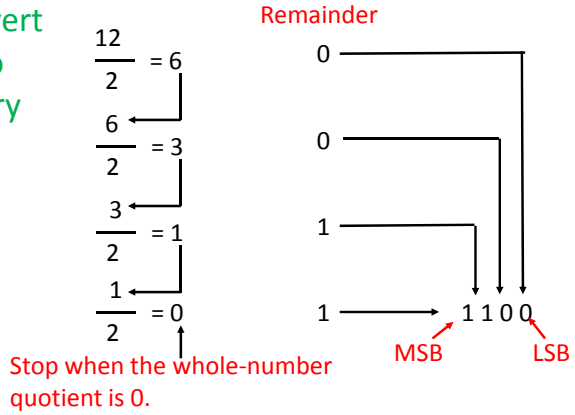
- To convert a decimal number to any base, R
- Generated digits are right to left, least to most

```
number = decimal number;
while (number > 0) {
  rem = number % R;
  Next digit is rem;
  number = number / R;  // integer division
}
```

## Decimal-to-Binary Conversion

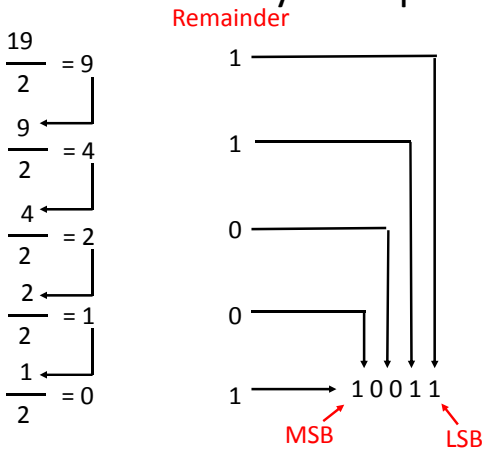
Repeated Division-by-2 Method:

Convert 12 to binary



## Decimal-to-Binary Example

Convert 19 to binary




## Converting Decimal to Hex

- Start with the largest even power of 16 bigger than the decimal number
- Repeat for all powers of 16, big to small

```
if the number >= power of 16 {
    quotient = number / power of 16;
    append quotient to result;
    number = number % power of 16;
}
Add the remainder to the result;
```

1048576
65536
4096
256
16

## Decimal to Hex Example

- Convert 317 to Hex
- The answer is

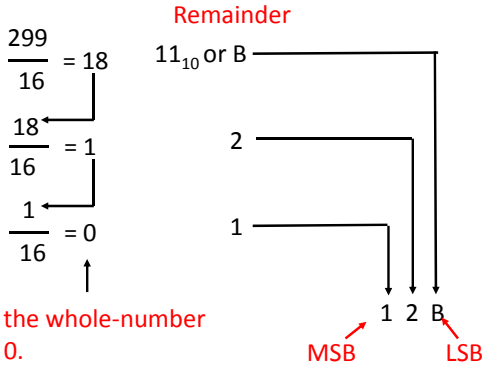
13D

| | | |
|---|---|---|
| 1048576 | 0 | 317 |
| 65536 | 0 | 317 |
| 4096 | 0 | 317 |
| 256 | 1 | 317/256 = 1, rem=61 |
| 16 | 3 | 61/16=3, rem=13 |
| 1 | D | |

## Decimal-to-Hex Conversion

Repeated Division-by-16 Method:

Convert 299 to hex.




Stop when the whole-number quotient is 0.

## Binary-to-Decimal Conversion

•For each 1 bit, add the value of that bit.

Convert the binary whole number 1101101 to decimal

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Weight: | $2^6$ | $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
| Binary number: | 1 | 1 | 0 | 1 | 1 | 0 | 1 |

$1101101 = 2^6 + 2^5 + 2^3 + 2^2 + 2^0$

$= 64 + 32 + 8 + 4 + 1$

$= 109$

| Decimal | Binary | Hexadecimal |
|---|---|---|
| 0 | 0000 | 0 |
| 1 | 0001 | 1 |
| 2 | 0010 | 2 |
| 3 | 0011 | 3 |
| 4 | 0100 | 4 |
| 5 | 0101 | 5 |
| 6 | 0110 | 6 |
| 7 | 0111 | 7 |

| Decimal | Binary | Hexadecimal |
|---|---|---|
| 8 | 1000 | 8 |
| 9 | 1001 | 9 |
| 10 | 1010 | A |
| 11 | 1011 | B |
| 12 | 1100 | C |
| 13 | 1101 | D |
| 14 | 1110 | E |
| 15 | 1111 | F |

## Online Quiz

- Complete the online quiz on base number conversion on the assignments page of the web site, http://williams.comp.ncat.edu/comp370/
- Due by 5:00pm on Friday, January 16, 2009
- Your score is the percentage right minus the number of seconds over 90.
- You may take the exam as often as you like. Your grade will be determined by your best score.

*This is not easy.*

## Converting Hex to Binary

- Each hexadecimal digit converts to four bits.
- Convert 2A7C to binary

| 2 | A | 7 | C |
|---|---|---|---|
| 0010 | 1010 | 0111 | 1100 |

## Converting Binary to Hex

- Group the binary bits into groups of four.
- Each group of four bits converts to a single hexadecimal digit.
- Convert 1101000101101 to hexadecimal

| 1 | 1010 | 0010 | 1101 |
|---|---|---|---|
| 1 | A | 2 | D |

## Decimal Fractions

- To the right of the decimal or radix point, each digit is 1/10$^{th}$ of the digit to the left.

| 10's | 1's | . | 1/10's | 1/100's | 1/1000's |
|---|---|---|---|---|---|
| 1 | 2 | . | 3 | 7 | 5 |

## Binary Fractions

- To the right of the radix point, each digit is ½ of the digit to the left.
- The decimal number 12.375 is

| 8's | 4's | 2's | 1's | . | 1/2's | 1/4's | 1/8's |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | . | 0 | 1 | 1 |

## Example Binary Fractions

| Decimal | Binary |
|---|---|
| 0.5 | 0.1 |
| 0.25 | 0.01 |
| 0.125 | 0.001 |
| 0.75 | 0.11 |
| 0.1 | 0.00011001100110011… |

## Binary-to-Decimal Conversion

Convert the Fractional binary number 0.1011 to decimal

Weight: $2^{-1}$ $2^{-2}$ $2^{-3}$ $2^{-4}$

Binary number: 0.1 0 1 1

$0.1011 = 2^{-1} + 2^{-3} + 2^{-4}$

$= 0.5 + 0.125 + 0.0625$

$= 0.6875$

## Decimal to Binary Conversion

```
number = decimal fraction;
while (number > 0) {
  number = number * 2;
  if ( number >= 1) {
     append 1 to result;
     result = result – 1;
  } else {
     append 0 to result;
  }
}
```

## Decimal to Binary Example

Convert 0.3125 to binary

| | |
|---|---|
| 03.125 * 2 = 0.625 | result = 0.0 |
| 0.625 * 2 = 1.25 | result = 0.01 |
| 0.25 * 2 = 0.50 | result = 0.010 |
| 0.5 * 2 = 1.00 | result = 0.0101 |